Georgia Discrete Analysis Conference

Abstracts for Monday May 14th

All talks are in Room 328 of the Boyd Graduate Studies Research Center

10:15-11:00

Misha Rudnev

Point-plane incidence theorem and some applications in positive characteristics

Abstract

The talk will sketch how the point-plane incidence theorem follows from one of the Guth-Katz theorems, and how it in turn implies the Stevens-de Zeeuw point-line incidence theorem in the plane. Then it will brush on on some geometric (that is non sum-product type) applications where progress has been made owing to the theorem so far. These are the finite field version of the Erdős distance problem in 3D and 2D, the problem of the minimum number of distinct values of a bilinear form in 2D, and the Fourier extension/restriction problem on the discrete paraboloid – over the prime field – in 3D and 4D. It is the latter topic that the more technical part of the talk is going to focus on: in a recent paper with Shkredov we prove an optimal $L^2 \to L^3$ Fourier extension result for the paraboloid in 4D and improve the known extension exponent from L^2 on the paraboloid in 3D from 68/19 to to 32/9.

11:10-11:55

Jozsef Solymosi

Sums and products along edges of a graph

Abstract

In their seminal paper Erdős and Szemerédi formulated conjectures on the size of sumset and productset of integers. The strongest form of their conjecture is about sums and products along the edges of a graph. In this talk we show that this strong form of the Erdős-Szemerédi conjecture does not hold. We give upper and lower bounds on the cardinalities of sumsets, product sets and ratio sets along graphs. (Joint work with Noga Alon and Imre Ruzsa)

2:00-2:45

Tom Sanders

The Erdős-Moser sum-free set problem I

Abstract

We consider the problem of finding a large subset S of a given set A of integers such that $s + s' \notin A$ whenever $s \neq s'$ and $s, s' \in S$. A greedy argument gives a logarithmic bound – $|S| \geq c \log |A|$ – and about 10 years ago Sudakov, Szemerédi and Vu made an important breakthrough when they showed one could take $|S|$ to be super-logarithmic. In these talks we shall give an argument to show one can take $|S| = \log^{1+c} |A|$ and sketch some limitations of the method.

The first talk will provide context and relate the problem to a local variant. In the second we shall sketch the proof of this local variant which has a convenient model formulation.

3:20-4:05

Freddie Manners

A proof of the inverse theorem for the Gowers norms with bounds

Abstract

The inverse theorem for the Gowers U^{s+1}-norm (say, over cyclic groups) is a key ingredient of a number of recent developments in additive combinatorics; most famously in work of Green and Tao on linear configurations in primes.

However, all known proofs of this result for $s > 3$ (Green–Tao–Ziegler; Szegedy / Anatolín Camarena–Szegedy) are highly infinitary and offer no effective bounds (and for $s = 3$, only terrible effective bounds).

We outline a new proof of this result for general s, which is finitary in flavor and does give non-terrible bounds. The key ingredient is a strong classification of "approximate polynomials", meaning functions $f : H \to \mathbb{R}$ for H some abelian group, whose $(s+1)$-st discrete derivatives are zero a positive proportion of the time, which could be of independent interest.

This is complementary to recent work of Gowers and Milicevic proving effective bounds for the inverse theorem over finite fields (when $s = 3$).

Georgia Discrete Analysis Conference

Abstracts for Tuesday May 15th

All talks are in Rooms 328 of the Boyd Graduate Studies Research Center

10:15-11:00

Tamar Ziegler

An interplay between additive combinatorics and algebraic geometry I

11:10-11:55

Pablo Candela

On cubic couplings and their applications

Abstract

I shall discuss recent joint work with Balázs Szegedy, in which we introduce cubic couplings as measure-theoretic objects that enable a unified treatment of several questions related to uniformity seminorms, in arithmetic combinatorics and in ergodic theory.

2:00-2:45

Tom Sanders

The Erdős-Moser sum-free set problem II

Abstract

We consider the problem of finding a large subset S of a given set A of integers such that $s + s' \notin A$ whenever $s \neq s'$ and $s, s' \in S$. A greedy argument gives a logarithmic bound – $|S| \geq c \log |A|$ – and about 10 years ago Sudakov, Szemerédi and Vu made an important breakthrough when they showed one could take $|S|$ to be super-logarithmic. In these talks we shall give an argument to show one can take $|S| = \log^{1+c} |A|$ and sketch some limitations of the method.

The first talk will provide context and relate the problem to a local variant. In the second we shall sketch the proof of this local variant which has a convenient model formulation.

3:20-4:05

David Conlon

Hypergraph expanders from Cayley graphs

Abstract

Given a k-uniform hypergraph H, consider the associated graph whose vertex set consists of the (k-1)-tuples of vertices in V(H) which are contained in some edge and where two vertices are joined if there is an edge of H containing both. We say that H is an expander if this associated graph is an expander. Using Cayley graphs over elementary abelian 2-groups, we give a randomisable construction of such expanders whose degree is polylogarithmic in the number of vertices.

Partly based on joint work with Jonathan Tidor and Yufei Zhao.

4:15-5:00

Jacob Fox

Tower-type bounds for Roths theorem with popular differences

Abstract

A famous theorem of Roth states that for any $\alpha > 0$ and n sufficiently large in terms of α, any subset of $\{1, \ldots, n\}$ with density α contains a 3-term arithmetic progression. Green developed an arithmetic regularity lemma and used it to prove that not only is there one arithmetic progression, but in fact there is some integer $d > 0$ for which the density of 3-term arithmetic progressions with common difference d is at least roughly what is expected in a random set with density α. That is, for every $\epsilon > 0$, there is some $n(\epsilon)$ such that for all $n > n(\epsilon)$ and any subset A of $\{1, \ldots, n\}$ with density α, there is some integer $d > 0$ for which the number of 3-term arithmetic progressions in A with common difference d is at least $(\alpha^3 - \epsilon)n$. We prove that $n(\epsilon)$ grows as an exponential tower of 2s of height on the order of $\log(1/\epsilon)$. We show that the same is true in any abelian group of odd order n. These results are the first applications of regularity lemmas for which the tower-type bounds are shown to be necessary. Joint work with Huy Tuan Pham and Yufei Zhao.

Georgia Discrete Analysis Conference

Abstracts for Wednesday May 16th

All talks are in Rooms 328 of the Boyd Graduate Studies Research Center

10:15-11:00

Tamar Ziegler

An interplay between additive combinatorics and algebraic geometry II

11:10-11:55

Sean Prendiville

Partition regularity and non-linear Diophantine equations

Abstract

Schur's theorem states that however one finitely colours the positive integers, there is always a solution to the equation x+y = z in which each variable receives the same colour. Rado completely characterised which linear equations possess this property and which do not. We discuss analogues of these results for certain non-linear Diophantine equations in sufficiently many variables.

2:00-2:45

Thomas Bloom

TBA

Abstract

3:20-4:05

David Conlon

Some problems of Burr and Erdős in additive number theory

Abstract

A sequence of positive integers A is said to be entirely Ramsey complete if, for any two-colouring of A, every positive integer can be written as the sum of distinct elements of A of the same colour. We show that there exists a constant C and an entirely Ramsey complete sequence A such that $|A \cap [n]| \leq C \log^2 n$ for all n. This is best possible up to the constant and solves a problem of Burr and Erdős. We also discuss several related problems stated by the same authors.

Joint work with Jacob Fox.

4:15-5:00

Jacob Fox

Removal lemmas

Abstract

The triangle removal lemma of Ruzsa and Szemerédi states that every graph on n vertices with $o(n^3)$ triangles can be made triangle-free by removing $o(n^2)$ edges. This fundamental result has many applications in mathematics and computer science. In this talk I will survey various removal lemmas and quantitative aspects.

Georgia Discrete Analysis Conference

Abstracts for Thursday May 17th

All talks are in Rooms 328 of the Boyd Graduate Studies Research Center

10:15-11:00

Olof Sisask

A notion of VC-dimension for subsets of groups – analysis, combinatorics and structure

Abstract

The VC-dimension of a family sets, named after Vapnik and Chervonenkis, is an interesting combinatorial concept with wide applicability. In this talk we shall discuss a notion of VC-dimension for subsets of groups and discuss the structure of subsets of abelian groups with bounded VC-dimension – illustrating what might be termed an arithmetic regularity lemma – as well as some analytic results about convolutions. These results can be used to give strong answers to classical additive combinatorial questions, such as finding structures in sumsets, in the bounded-VC setting.

11:10-11:55

Jozsef Solymosi

Applications of hypergraph removal lemmas in arithmetic settings

Abstract

When the vertices of a hypergraph are elements of a group and the edges are defined using arithmetic relations then it has a very special structure. I will show examples when using hypergraph removal lemmas in such graphs one can prove results stronger than in general graphs. For example let us suppose that the edges in a 3-uniform hypergraph, H_n, are defined as triples $(a, b, a * b)$, where a, b are from a finite group of n elements. Then for every integer d, there is a k, such that any subgraph of H_n with at least cn^2 edges contains k vertices spanning at least $k + d$ edges. (provided that n is larger than a threshold $m = m(c)$). The Brown-Erdős-Sos conjecture states that if a 3-uniform hypergraph, H_n, doesn't have $k + 3$ vertices spanning at least k edges the the number of edges in H_n is $o(n^2)$. The only known case of the conjecture is $k = 3$, this is equivalent to the triangle removal lemma of Ruzsa and Szemerédi.